

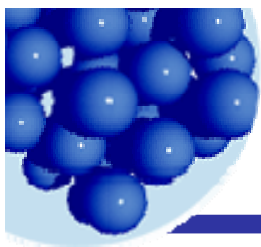
# Scaling relations

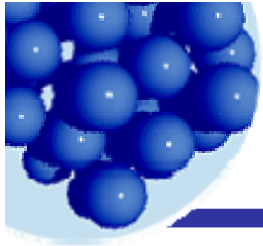
Dispersions in liquids: suspensions,  
emulsions, and foams

*ACS National Meeting*

*March 21 – 22, 2009*

*Salt Lake City*





## Perikinetic flocculation: Collisions by Brownian motion (1)

$J_{ij} = k_{ij}n_i n_k$  is the number of collisions per unit volume and time

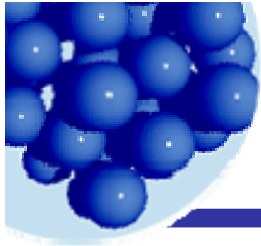
$$k_{ij} = \frac{2kT}{3\eta} \frac{(a_i + a_j)^2}{a_i a_j} \text{ second order rate constant}$$

where  $kT$  is the Boltzmann factor

$\eta$  is the viscosity

$a_i$  and  $a_j$  are the particle radii

Different sized  
particles flocculate  
faster.



## Perikinetic flocculation: Collisions by Brownian motion (2)

For equal sized particles and interaction potential  $U$ :

$$n = \frac{n_0}{1 + \frac{4kT}{3\eta W} n_0 t}$$

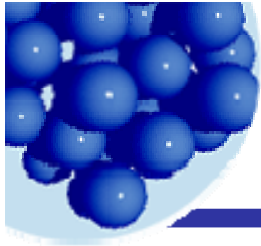
is the particle concentration

$$t_{1/2} = \frac{3\eta W}{4kT n_0} = \frac{\eta \pi a^3 W}{\Phi kT}$$

is the half-life

$$W = 2a \int_0^{\infty} \exp\left(\frac{U(H)}{kT}\right) \frac{dH}{H^2}$$

is the *Stability Ratio*



## Orthokinetic agglomeration: Collisions by fluid motion

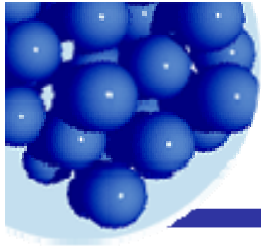
With laminar flow,  
shear rate =  $G$

$$n = n_0 \exp\left(\frac{-4G\Phi}{\pi} t\right)$$

For turbulent flow,  
an model based on  
power input gives:

$$\bar{G} = \sqrt{\frac{\varepsilon\rho}{\eta}}$$

where  $\varepsilon$  is the power input per unit mass  
 $\rho$  is the fluid density  
 $\eta$  is the fluid viscosity



## A comparison of rates (1)

Perikinetic  $k_{ij} = \frac{2kT}{3\eta} \frac{(a_i + a_j)^2}{a_i a_j}$

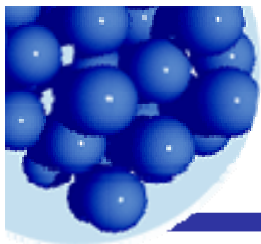
Orthokinetic  $k_{ij} = \frac{4}{3} G (a_i + a_j)^3$

Differential settling  $k_{ij} = \left( \frac{2\pi g}{9\eta} \right) (\rho_s - \rho) (a_i + a_j)^3 (a_i - a_j)$

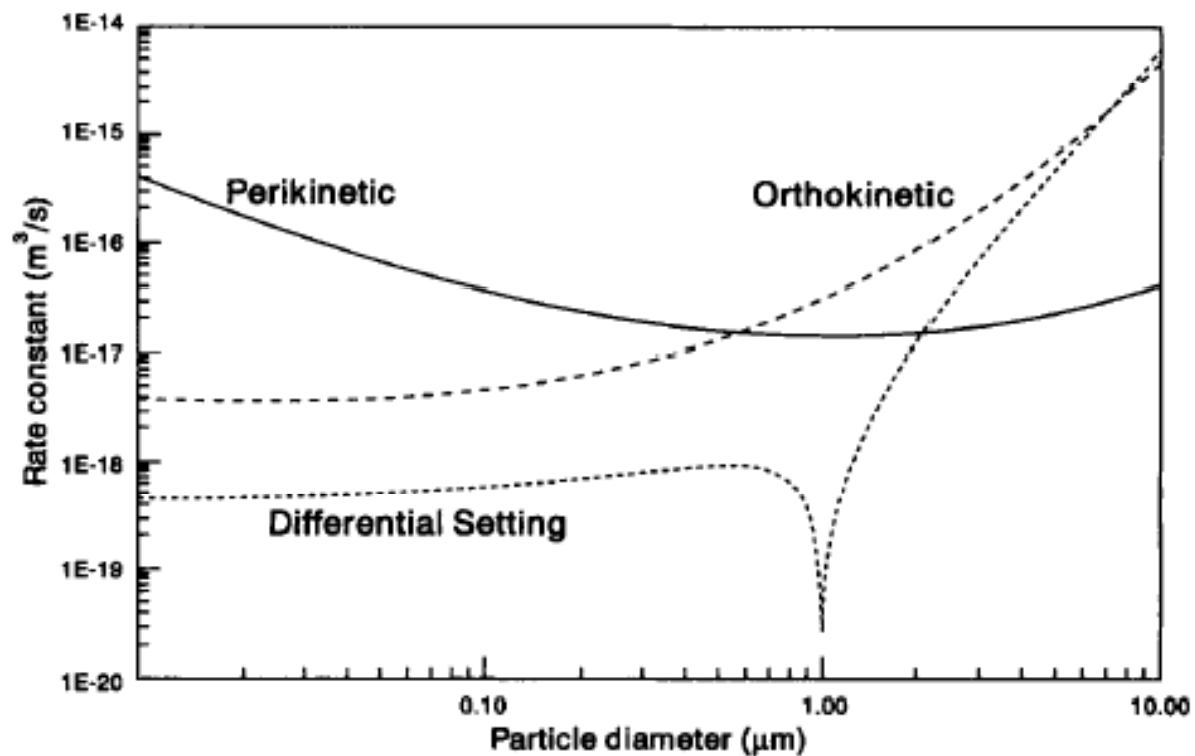
where  $g$  is gravitational acceleration

$\rho_s$  is the particle density

$\rho$  is the liquid density

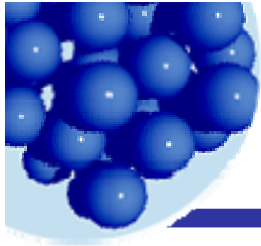


## A comparison of rates (2)



**Figure 6.3** Comparison of collision rate constants for different transport mechanisms (see text)

Elimelech, p. 170.



# Characteristic times and forces

## Characteristic times (seconds)

Force	Time scale	Sphere radius	
		$10^{-2} \mu\text{m}$	$1 \mu\text{m}$
Brownian	$a^2 / D$	$10^{-6}$	$10^{-2}$
Viscous	$\rho a^2 / \eta$	$10^{-10}$	$10^{-6}$
Convection	$a / U$	$10^2$	1

## Characteristic forces ( $\text{N/m}^2$ )

Force	Force scale	Sphere radius	
		$10^{-2} \mu\text{m}$	$1 \mu\text{m}$
Brownian	$kT/D$	$10^{-13}$	$10^{-15}$
Viscous	$6\pi\eta aU$	$10^{-20}$	$10^{-14}$
Dispersion	$A/a$	$10^{-12}$	$10^{-14}$
Electrostatic	$\epsilon\epsilon_0\zeta_0^2$	$10^{-12}$	$10^{-12}$

$a$  = sphere radius  
 $D$  = Diffusion coefficient  
 $\rho$  = density  
 $\eta$  = viscosity  
 $U$  = fluid velocity  
 $kT$  = Boltzmann factor  
 $A$  = Hamaker constant  
 $\epsilon$  = dielectric constant  
 $\epsilon_0$  = permittivity of free space  
 $\zeta_0$  = zeta potential

Russel, pp 9-12.