The flow of Herschel-Bulkley fluids

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Thin polymer-filled layers coated from dispersions
Typical shear thinning rheogram

Photoconductive Pigment in Nonaqueous Polymer Solution

![Graph showing shear thinning behavior](graph.png)
Data is reasonably fitted by a power law

\[ \eta = \eta_0 \dot{\gamma}^{n-1} \]

\( n \) is the power law index and is one for a Newtonian fluid.

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Photoconductive Pigment in Nonaqueous Polymer Solution

Shear rate (s\(^{-1}\))

Viscosity (mPa s)

\( \eta = 0.58 \dot{\gamma}^{-0.75} \)

\( n = 0.25 \)
The flow of a shear thinning liquid through a pipe

The reduced flow rate of a shear thinning liquid is:

\[ \bar{v} = \frac{1 + 3n}{1 + n} \left[ 1 - \left( \frac{r}{R} \right)^{\frac{1}{n+1}} \right] \]

Same data replotted

Shear stress versus shear rate

Shear stress (Pa)

Shear rate (s^{-1})
How yield points approximate power law flow

Shear stress versus shear rate

The slope is the viscosity.
Herschel-Bulkley rheology

Model the flow as shear thinning above some yield point:

\[ \tau = \tau_0 + m\dot{\gamma}^n \]

Note the value of “n” is now higher, apparently less shear thinning.
Effect of a yield point on flow

If a fluid has a yield point, $\tau_0$, and is under pressure, $p$, a column of radius $r$ and length $L$ in the center moves as a plug. The plug radius can be calculated.

$$p = \frac{\tau_0 (2\pi r L)}{\pi r^2}$$

or

$$r = \frac{2\tau_0 L}{p} = \frac{2\tau_0}{P}$$

$P$ is the pressure drop.
Flow of an Herschel-Bulkley fluid through a pipe

\[ r > \frac{2\tau_0}{P} \]

\[ \text{velocity} = \frac{-P^2 (3n+1)(2n+1)R^2 ((PR - \tau_0)^{-1/n} (Pr - \tau_0)^{\frac{1+n}{n}} - PR + \tau_0)}{2n^2 \tau_0^3 - R^3 P^3 + 2n\tau_0^2 RP + \tau_0 R^2 P^2 n - 2R^3 P^3 n^2} \]

else

\[ \text{velocity} = \frac{2\left(1 - \frac{\tau_0}{RP}\right)^2}{\left(1 - \frac{4}{3} \frac{\tau_0}{RP} + \frac{1}{3} \left(\frac{\tau_0}{RP}\right)^4\right)} \]
Flow of Herschel Bulkley fluid

The fitted Herschel-Bulkley parameters:

\[ n = 0.75 \]
\[ \tau = 0.5 \text{ Pa} \]

with a reasonable pressure drop

\[ P = 10^6 \text{ Pa}/0.1m \]

Note: The radius of the plug is:

\[ r = \frac{2 \times 0.5 \text{ Pa}}{10^5 \text{ Pa}/0.1m} = 1\mu m \]
Comparison of predicted flows: Power law and Herschel-Bulkley
Some Problems Require More Study
Surface Energy Characterization of Fibers, Fillers and Paper

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Philadelphia, August 22-26, 2004
228th ACS National Meeting
Pennsylvania Convention Center, Philadelphia

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